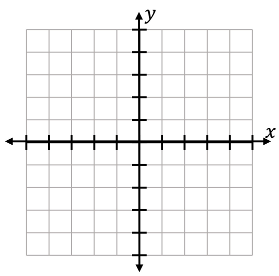
AP CALCULUS AB SUMMER PACKET

Section 1: Graphing Functions

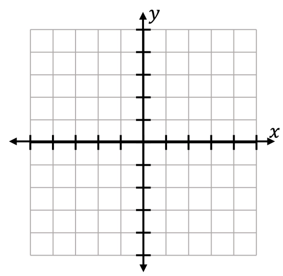
- You should know the parent functions including quadratic, cubic, square root, rational, and trig
- You should be able to graph piecewise functions
- You need to know these parent functions WITHOUT examples!

Graph each of the following without a calculator.

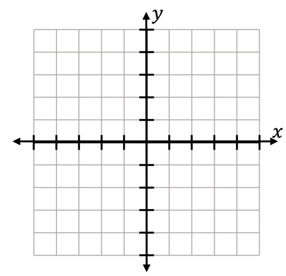
$y = x^2$



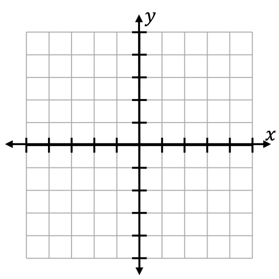
$y = x^3$



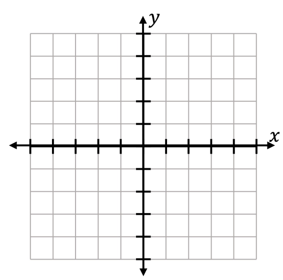
$y = \sqrt{x}$



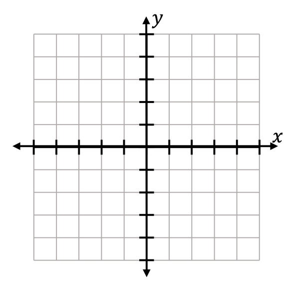
$y = e^x$



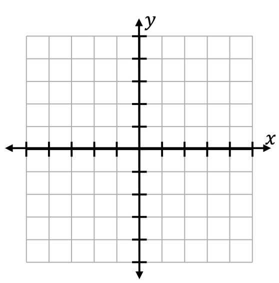
$y = \ln x$



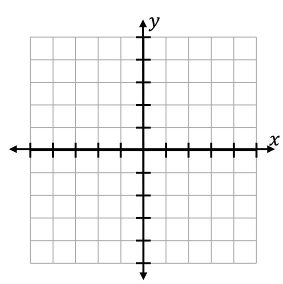
$y = \dfrac{1}{x}$



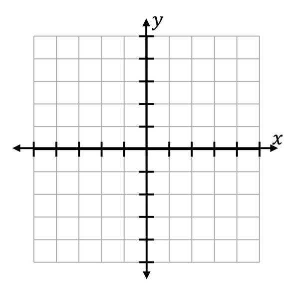
$y = \dfrac{1}{x^2}$



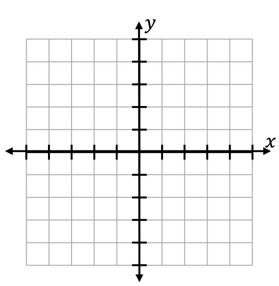
$$y = \begin{cases} -1, \text{ if } x \le -1 \\ 3x + 2, \text{ if } -1 < x < 1 \\ 7 - 2x, \text{ if } x \ge 1 \end{cases}$$



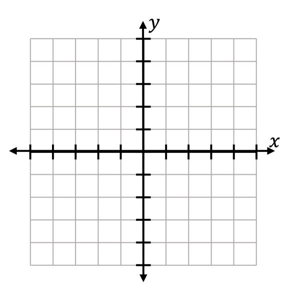
$$y = \begin{cases} x^2 + 1, \text{ if } x > 0 \\ -2x + 2, \text{ if } x \le 0 \end{cases}$$



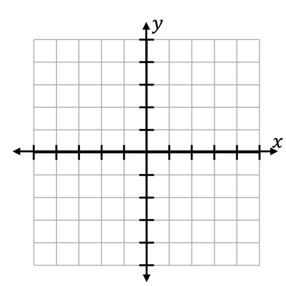
$y = \sin x, -2\pi \le x \le 2\pi$



$y = \cos x, -2\pi \le x \le 2\pi$



$y = \tan x, -2\pi \le x \le 2\pi$



Section 2: Lines

- Slope: ______________________________
- Point-Slope Form: _____________________
- Slope-Intercept Form: _________________
- Standard Form: _______________________
- Parallel Lines: ________________________
- Perpendicular Lines: __________________

  ____
- X-Intercept: _________________________
- Y-Intercept: _________________________

Find the slope between (6, 10) and (-3, 5)

Write the equation of the line parallel to $y = -2x + 5$ that passes through the point (4, 1).

Write the equation of the line perpendicular to $y = 3x - 6$ that passes through the point (-6, 4).

Section 2 Practice

1. Find the slope passing through (4, 5) and (-2, 12).
2. Find the x- and y-intercepts of the line $2x + 5y = 15$.
3. Write the equation of the line in point-slope form that passes through the points (-3, 2) and (4, 7)
4. Write the equation of the line parallel to $y = 3x + 5$ passing through (-9, 15).
5. Write the equation of the line perpendicular to $y = -4x + 3$ passing through (16, -5).

Section 3: Algebraic Expressions and Functions

- Factor Polynomials:
  - $6x^2 + 7x - 3$
  - $8x^3 - 12x^2 + 2x - 3$
- Solve by Factoring:
  - $6x^2 + 6 = 5x + 10$
  - $8x^3 - 27 = 0$
- Evaluate $f(x) = 3x^2 - 5x + 4$ at $x = 2$.
- Composite Functions: Let $f(x) = 2x + 5$ and $g(x) = x^2 - 4$.
  - Find $f(g(3))$
  - Find $f \circ g(-1)$
  - Find $g \circ f(x)$

Section 3 Practice

1. Solve: $2x^3 - 8x^2 + 3x - 12 = 0$
2. $3x^5 - 33x^3 = -84x$
3. $27x^3 + 343 = 0$
4. $9x^3 - 54x^2 - 4x = -24$

Let $f(x) = 3x - 4$ and $g(x) = 2x^2 - 3$

5. $f(2) =$
6. $g(-4) =$
7. $f(x + 1) =$
8. $g(f(3)) =$
9. $f \circ g(2) =$
10. $f(g(x)) =$

Section 3: Algebraic Expressions and Functions (cont.)

- Inverse Functions:
- How do the graph of $f(x)$ and $f^{-1}(x)$ compare?
- If (3, 5) is on $f(x)$, then what point will be on the graph of $f^{-1}(x)$?
- Find the inverse of $f(x) = 3x - 4$

- Rational Exponents and Rational Expressions:
  - Rewrite as a rational expression: $\sqrt[4]{x^5}$
  - Rewrite as a radical expression: $\frac{2}{x^{5/3}}$
  - Simplify: $\frac{(x^2)^4 x}{x^{10}}$
  - Simplify: $\sqrt{x} \cdot \sqrt[3]{x} \cdot x^{\frac{1}{5}}$
  - Simplify: $9^{\frac{3}{2}}$

Section 3 Practice (cont.)

1. If (-2, 4) is on the graph of $f(x)$, then what point is on the graph of $f^{-1}(x)$?
2. Find the inverse of $f(x) = \frac{x^2}{2} - 5$
3. Find the inverse of $f(x) = \frac{2}{3}x + 6$
4. Rewrite using rational exponents: $\sqrt[4]{x^3} \cdot \sqrt{x^5}$
5. Simplify: $8^{-\frac{2}{3}}$

Section 3: Algebraic Expressions and Functions (cont.)

- Simplify: $\frac{x}{x-5}+\frac{3}{x+9}$

- Simplify: $\frac{3(x+h)^2-3x^2}{h}$

- Simplify: $\frac{\frac{1}{x}+\frac{3}{x^2}}{4-\frac{1}{x}}$

- Solve: $\frac{x-4}{x}=\frac{5}{2x+3}$

- Find all values where the following expression is equal to 0 or undefined: $\frac{4x-1}{(x-2)(x^2+3)}$

Section 3 Practice (cont.)

1. Simplify: $\frac{x}{x-2}-\frac{3}{x+7}$

2. Simplify: $\frac{2(x+h)^2-5-(2x^2-5)}{h}$

3. Simplify: $\frac{\frac{3}{x}-\frac{2}{x^2}}{4-\frac{1}{x}}$

4. Solve: $\frac{2x}{x-2}=\frac{3x+5}{6}$

5. Find all values where the following expression is equal to 0 or undefined: $\frac{3x+5}{(x-1)(x^4+7)}=0$

Section 4: Exponentials and Logarithms

- Properties of Exponents:
  - $x^m \cdot x^n = x^{m+n}$
  - $\frac{x^m}{x^n} = x^{m-n}$
  - $(x^m)^n = x^{m \cdot n}$
  - $x^0 = 1$
  - $x^{-m} = \frac{1}{x^m}$
- Properties of Logarithms:
  - $\ln(ab) = \ln(a) + \ln(b)$
  - $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
  - $\ln(a^n) = n \ln(a)$
- Change of Base: $\log_b x = \frac{\ln x}{\ln b}$ **we will mostly use the natural log, which is log base e, so you must know this change**

- Expand: $\ln\frac{x^3y^2}{z^5}$
- Condense: $3\ln x + 4\ln y - \frac{1}{2}\ln z$
- Simplify:
  - $4\ln e^5$
  - $e^{\ln 9}$
- Solve: $\ln(x-1) = 5$
- Solve: $e^{2x} = 6$

Section 4 Practice

1. Simplify:
   a. $9 \ln e^7$
   b. $e^{\ln 5}$
   c. $\ln 1$
   d. $\ln e$
   e. $\ln 25 + \ln 4$
   f. $\ln 40 - 2 \ln 5$
   g. $\log_2(\sqrt{2})^5$
   h. $\ln \sqrt[3]{e}$

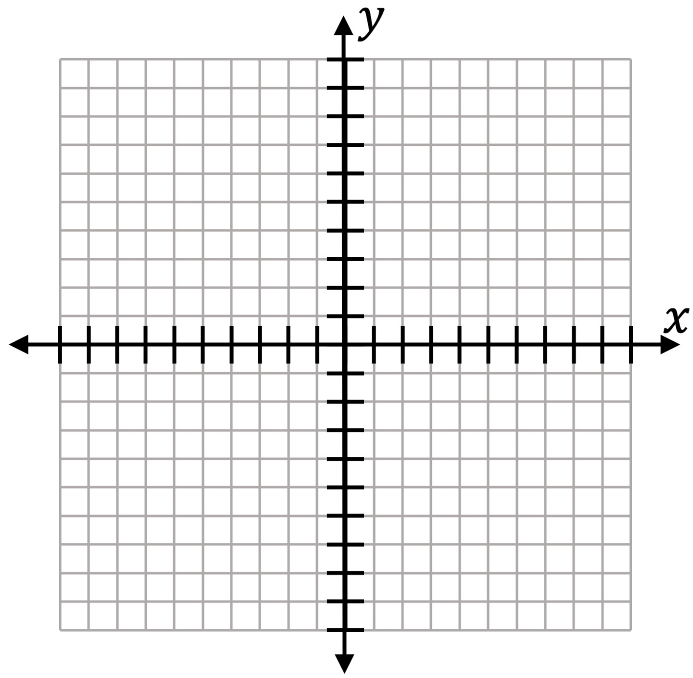
2. Expand:
   a) $\ln\frac{2x^7}{y^3}$
   b) $\ln\frac{\sqrt{x}}{y^3z}$
   c) $\ln\sqrt{\frac{x}{y^5z^4}}$

3. Solve:
   a) $-\ln(x-1) = 3$
   b) $\ln x = 3$
   c) $e^{2x} = 19$

Section 5: Graphing Rational Functions

- Vertical Asymptotes and Holes:
  - $f(x) = \dfrac{x+4}{x^2(x-2)}$
  - $f(x) = \dfrac{x-1}{x^2+3x-4}$
- Horizontal Asymptotes:
  - If the degrees of the numerator and denominator are equal, then the HA is at ______________________________
  - If the degree of the denominator is greater than the degree of the denominator, then the HA is ________
  - If the degree of the numerator is greater than the degree of the denominator, there is ___ HA
    - If the degree of the numerator is exactly one greater than the degree of the numerator, then there is a ___________________

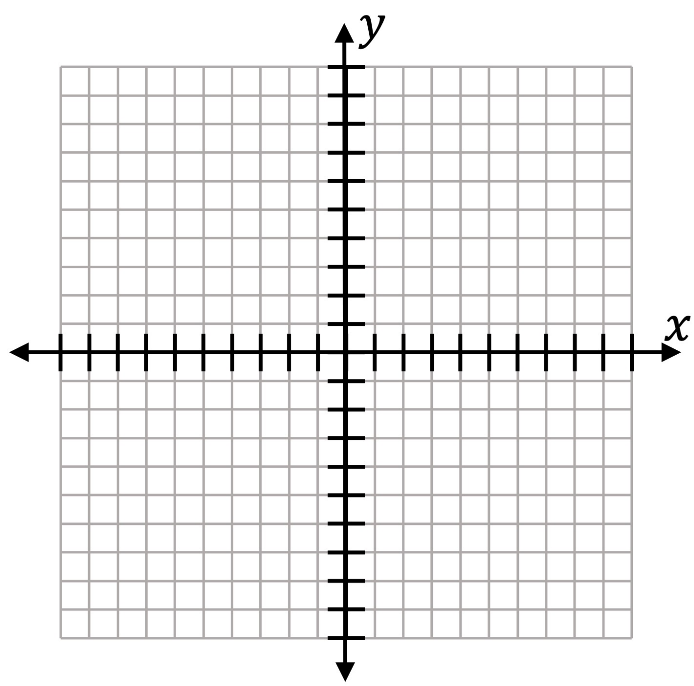
- Graph: $f(x) = \dfrac{x-2}{x^2-4}$



Section 5 Practice

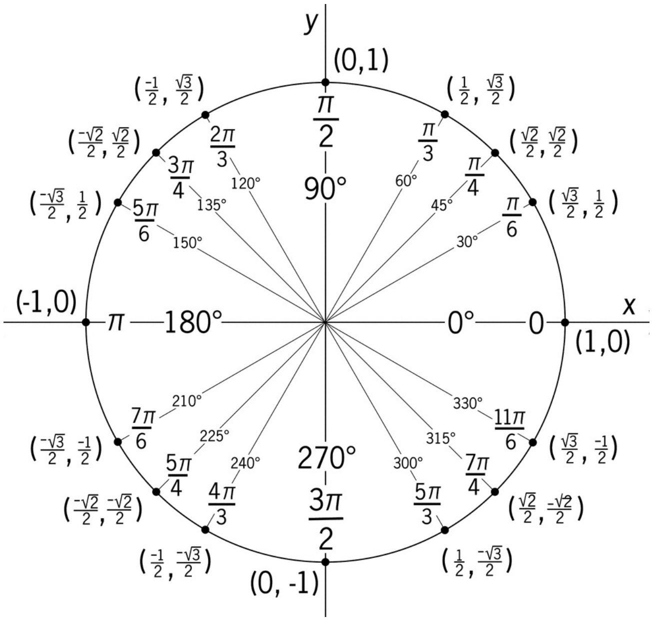
1. Find all asymptotes and discontinuities:
   a. $f(x) = \dfrac{x-3}{x^2+x-12}$
   b. $f(x) = \dfrac{x^2-25}{5-x}$
   c. $f(x) = \dfrac{2x^2+x-3}{x^2+2x-3}$
   d. $f(x) = \dfrac{x^3-8}{x^2+3x-10}$
   e. $f(x) = \dfrac{(2x-3)^2}{x^2-2x}$

2. Graph: $f(x) = \dfrac{x^2+8x+15}{x^2-9}$



Section 6: Trigonometry

- You should be able to evaluate trig AND inverse trig functions using the unit circle or special right triangles WITHOUT a calculator
- $\sin^2 x + \cos^2 x = 1$
- $\sin 2x = 2\sin x\cos x$
- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\sin(-x) = -\sin x$ • $\cos(-x) = \cos x$ • $\tan(-x) = -\tan x$



- Solve: $3\cot^2 x - 1 = 0$
- Solve: $2\sin^2 x - \sin x = 0$
- Solve: $\cos^2 x = 1 - \sin x$
- Solve: $\csc x + \cot x = 1$

Section 6 Practice

1. Evaluate the following:
   a. $\tan\frac{\pi}{3}$
   b. $\cos\frac{\pi}{2}$
   c. $\sin\frac{3\pi}{4}$
   d. $\sec\frac{\pi}{6}$
   e. $\cot\frac{7\pi}{6}$
   f. $\csc\frac{\pi}{2}$
   g. $\sin^{-1}(0)$
   h. $\cos^{-1}\frac{1}{\sqrt{2}}$
   i. $\arctan\frac{\sqrt{3}}{3}$
   j. $\text{arcsec}\,\frac{2}{\sqrt{3}}$
   k. $\csc^{-1}(-2)$
   l. $\text{arccot}\,\frac{\sqrt{3}}{3}$

2. Solve the following:
   a. $2\cos x = \sqrt{3}$
   b. $\sin(3x) = 0$
   c. $2\cos(2x) = 0$
   d. $1 - \sin^2 x = \frac{1}{4}$